A Ph.D. SYNOPSIS

In the area of

Computers & Soft Computing

On the topic of

Inverted Pendulum Control Using Soft Computing Techniques

Submitted by

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May 2012

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1. Introduction

The inverted pendulum is a classic problem in dynamics and control theory and widely used as benchmark for testing control algorithms (PID controllers, neural networks, genetic algorithms, etc). The inverted pendulum control problem is the same as trying to balance a broom handle on your fingertip. An inverted pendulum consists of a thin rod attached at its bottom to a moving cart and the base of the handle is moved to attempt to keep it upright.

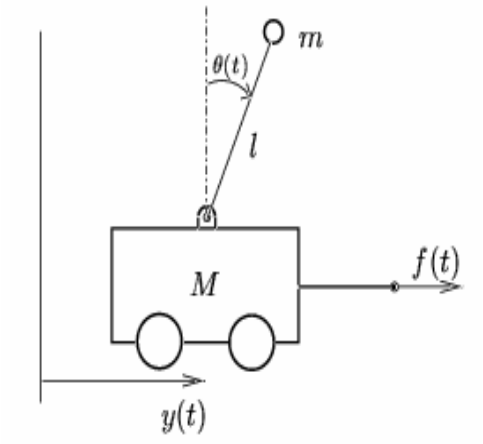
The inverted pendulum is related to rocket or missile guidance, where thrust is actuated at the bottom of a tall vehicle. The understanding of a similar problem is built in the technology of Segway, a self-balancing transportation device. The largest implemented uses are on huge lifting cranes on shipyards. Through continuous research new control methods have been developed and then applied to high-tech industries such as aeronautical engineering and robotics. In addition to educational purposes, an inverted pendulum is also an excellent tool for researchers of modern control theories.

Several mechanical configurations have been studied, and many controller types have been used to achieve the balancing of an inverted pendulum, to swing one from the stable state to the unstable (inverted condition), or even to keep the pendulum at some arbitrary angle of inclination.

Among the various techniques available, soft computing techniques are becoming useful in developing the methodologies for balancing inverted pendulum and understanding the dynamics of control. It is proposed to develop a suitable approach using soft computing techniques (ANN, FS, and GA) and quantum computing for the inverted pendulum control.

2. Modeling of Inverted Pendulum

The pendulum system is one of the classical examples used in connection with feedback control. The Inverted Pendulum (IP) consists of a cart confined to linear motion along a track and a rod attached to the cart by a hinge joint.



The cart is equipped with a motor that accepts a voltage and outputs a torque as a function of the voltage applied. Generally, the inverted pendulum system is modeled as a linear system and hence the modeling is valid only for small oscillations of the pendulum. The system is nonlinear, unstable, non minimum phase and under actuated. It is not full-state measurable with noisy measured signals. Because designing a nonlinear control system is rather complicated, engineers usually build a linearized model of inverted pendulum on the assumption that the pendulum does not incline far from the balanced position. A linear controller can stabilize the system around the balanced position if this model is properly built. However, it is only a nonlinear controller that can stabilize the system globally. State equations of an inverted pendulum are usually derived from the pendulum torque equation and horizontal Newton's law at the base that carries the pivot as equations for the pendulum angular velocity, the angular acceleration, the pivot velocity, and the pivot acceleration.

In general, the model of the inverted-pendulum system can be divided into three separate models: the inverted-pendulum model (the mechanical system of the cart and the pole), the dc-motor model, and the transmission model. During modeling, the control task is to swing up the pendulum from its natural pendant position and to stabilize it in the inverted position, once it reaches the upright equilibrium point. The cart must also be homed to a reference position on the rail. All this is achieved only by moving the cart back and forth within the limited cart travel along the rail.

Many parasitic effects exist such as friction, elastic modes of rod and shaft, backlash effects of gears and belts, together with input saturation. These make the inverted pendulum systems a classic tool in control engineering. It is used to illustrate ideas in nonlinear control, task-oriented control, hybrid systems and control of chaotic systems.

Friction in the motor cart system can be modeled with three components: viscous friction and static (dry) friction terms Coulomb friction and stiction. The physical phenomena in the inverted pendulum can be described with laws of dynamics and parameters can be determined using rulers, micrometers, scales and timer, modeling and the parameterization of damping. Initially only viscous friction is taken into account in developing equations of motion. This is sufficient for the pendulum pivot joints but not adequate to describe friction forces in the motor, timing belt transmission and between the track and cart.

Simple tests with the motor-cart system show that friction forces have a non-linear nature. For studying friction more precisely, the pendulum links can be removed from the cart. Thus the motor- cart can be handled mechanically as a one degree of freedom system.

The main contributors to the friction forces are the DC motor and pulley–belt-system that transfers torque from motor to force on the cart. The motor has friction between the brushes and commutator as well as in the bearings. The friction in the pulley–timing belt-cart transmission is more sophisticated. In both the systems, in addition to viscous friction, static friction terms like coulomb friction and stiction affect the control performance. The DC motor is the driving force for the cart at the base of leading screw and ball nut. The pendulum is located in the rolling bearings. In both systems viscous friction dependent on speed of rotation occurs. Relationship of viscous friction and speed of rotation is linear.

The inverted pendulum system belongs to the class of under-actuated mechanical systems having fewer control inputs than degrees of freedom. This renders the control task more challenging making the inverted pendulum system a classical benchmark for testing different control techniques.

While swinging up the pendulum, the energy of pendulum is driven toward a value equal to the steady-state upright position, and the pendulum-cart-movement restriction is considered. When the pendulum approaches the upright position, it must be caught with an appropriate strategy. It has been shown that a robust state controller must be designed to capture the pendulum in the upright position. While balancing the pendulum in the upright position with different state controllers, the pendulum stops at the upper steady state after adapting the state-controller gains. The adaptation method is based on the norm of the state vector. The real-time experiments have been very impressive even during the activity of defects or after changing the desired cart position. The fuzzy swinging algorithm swings the pendulum from any initial condition, and the pendulum cart always remains within the pendulum-rail limits

3. Literature review

The Inverted Pendulum problem is one of the most important problems in dynamics and has been studied excessively in the control literature. The reason behind such extensive studies of the pendulum relies behind the fact that many important engineering systems can be approximately modeled as pendula. For example, in thrust vectored rocket control, the pitch dynamics of a rocket can be approximated by a simple pendulum. In robot systems, the relation is pertinent with inverted pendulum systems [SPR 98]. In biomechanics, the pendulum is used to model bipedal dynamic walking [SHK 96]. The pendulums are also used in the study of wheeled motion [PTK 05] and balancing mechanisms [HSP 92]. A self balancing human transportation vehicle has been used as a pedagogical tool for teaching the concepts of feedback control to engineering students [LIS 09]. It is a benchmark problem that provides many challenging problems to check the design of newly developed control systems using different techniques. Being a benchmark problem in the nonlinear control realm, the inverted pendulum on a cart problem had mostly been used in simulations in the adaptive fuzzy control field. The system is nonlinear, unstable, non minimum phase and under actuated.

While controlling a real inverted pendulum, there are several limitations and constrains that are to be considered. Instead of driving the cart force (acceleration) directly, in particular case, the cart velocity is driven over dc-motor voltage, so we must consider the cart velocity limitations instead of the cart-acceleration limitations. Another very important limitation is that the rail has a limited length and, thus, the cart movement is limited. In the literature, there are several solutions for swing up and stabilization of a cart pendulum with a restricted travel.

There are a number of different versions of the inverted pendulum system offering a variety of interesting control challenges. The most common types are: the single rod on a cart inverted pendulum considered in [YOK 99], [AKJ 96], [OHA 95], the Double Inverted Pendulum (DIP) on a cart setup [ZHW 01], the rotational single-arm pendulum [FTK 91], [FTK 92] and the rotational two-link pendulum [YAM 94]. The control techniques involved are also numerous, ranging from simple conventional controllers to advanced and intelligent control techniques based on modern non-linear control theory.

Many control paradigms address the inverted pendulum on a cart problem as a fourth-order under actuated system. Examples include, experimental implementations using linear control techniques [IHN 01], linear parameter varying (LPV) techniques [KJH 99], the control under constraints [CVA 04], the grey prediction model [HSJ 00], and energy control using potential and kinetic energy [OTR 02]. In the sequel, we discuss the feasibility and implementation issues of applying adaptive fuzzy logic control (AFLC) to this problem. Wang presented a foundation of AFLC where it combines the merits of both conventional fuzzy and adaptive control. It provides online training techniques that can adapt the fuzzy controller parameters to different parameter variations and can identify a correct fuzzy rule base for controlling nonlinear systems. AFLC addresses the problem of controlling classes of unknown nonlinear systems using both indirect adaptive schemes [WLX 94], [EAL 04], [YOB 98], [LCG 01], and direct adaptive schemes [WLX 94] and [WCH 02]. Based on the second order model, this problem is used as a simulation example in [WLX 94], [EAL 04], [YOB 98], [KME 02] to demonstrate the performance of the adaptive controllers suggested therein.

Unfortunately, the second order model avoids an interesting control problem concerning the unstable zero dynamics associated with the cart sub system. Many of the adaptive fuzzy control schemes devised for the problem are based on feedback linearizing (FBL) control laws [KHK 02]. They address only the rod dynamics. Implementation is limited to simulation. In order to achieve feasible control, the devised scheme should address the systems as a whole, i.e., a fourth-order under actuated system. In [ODR 97], a reference is made to the unstable zero dynamics problem accompanying the FBL-based control schemes. However, it was possible to implement the controller since the model used was that of the rotary inverted pendulum. In [CCS 98], the authors present adaptive fuzzy control for the pole cart problem using sliding control input. The stability of the algorithm presented is based on choosing a certain form of the reference that leads, based on the linearized system model, to stable cart dynamics if the pole follows the reference. Here only regulation results are obtained based on the reference.

Barto et al. [BAG 83] demonstrated a quantization of the state space of the inverted pendulum with which single layer neural networks could learn to balance the pendulum. A

second solution is to add a second adaptive layer that learns such a representation. Anderson [ACW 87] extended the work of Barto et al. by applying a form of the popular error back-propagation method to two-layered neural networks that learn to balance the pendulum given the actual state variables of the inverted pendulum as input.

Fuzzy controllers have recently found many successful applications in both industry and household appliances. For complex and/or ill-defined systems that are not subdued to conventional control techniques, fuzzy logic control provides a feasible alternative since it can capture the approximate, qualitative aspect of human reasoning. However, the performance of fuzzy controllers depends on two important factors, i.e., the soundness of knowledge acquisition techniques and the availability of human experts. These two factors severely restrict the application domains of fuzzy controllers. We have proposed the architecture of ANFIS (Adaptive-Network-based Fuzzy Inference System)[JJS 91], [JNS 91], [JGS 91] which can bypass the first problem by eliciting the membership functions of fuzzy-if then rules directly from a desired input-output data set. The basics of adaptive networks and the ANFIS architecture are described in the next section.

In recent years, Takagi-Sugeno fuzzy models have received considerable interests [TKT 85], [TSC 02], [PCW 04], [LIJ 00] as a convenient and flexible tool for handling complex nonlinear systems, where their consequent parts are linear systems connected by IF-THEN rules. They can approximate nonlinear systems to any preciseness with dynamic combination of finite linear subsystems, and this allows designers to take advantage of conventional linear systems to analyze and design the nonlinear systems. Based on T-S fuzzy models, so-called parallel distributed compensation (PDC) is one significant control design framework [SGM 86], which can utilize Lyapunov function based control design techniques to the control synthesis problem. Direct Lyapunov method has been used for stabilization of Furuta pendulum and two wheeled pendulum [RMT 11], [AIC 10]. For attenuating external disturbances or/and system uncertainties, $H\infty$ schemes for fuzzy systems are discussed based on kinds of sufficient conditions of the quadratic stabilization[CSG 00], [LKR 01], [LXD 03], [ROD 10], the controllers which can

guarantee global stability and certain level performance can be derived by solving optimization or feasibility problems under a group of LMI conditions.

With the developments of neural networks and fuzzy systems, it is known that the qualitative knowledge of a system can also be represented in a nonlinear functional form. On the basis of this idea, some fuzzy models based fuzzy control system design methods have appeared in the fuzzy control field [CCL 93], [TKT 85], [TNK 92], [TNK 93], [TNK 94], [WHO 96]. These methods are conceptually simple and straightforward. Linear feedback control techniques can be utilized as in the case of feedback stabilization. The procedure is as follows. First, the nonlinear plant is represented by a Takagi–Sugeno-type fuzzy model [TSN 85]. In this type of fuzzy model, local dynamics in different state-space regions are represented by linear models. The overall model of the system is achieved by fuzzy "blending" of these linear models through nonlinear fuzzy membership functions. The control design is carried out on the basis of the fuzzy model via the so-called parallel distributed compensation scheme. The idea is that for each local linear model, a linear feedback control is designed. The resulting overall controller, which is nonlinear in general, is again a fuzzy "blending" of each individual linear controller. At the same time, the robust stabilization problem has been also considered in the case of Takagi–Sugeno (T–S) fuzzy model [TNK 96], [TNK 94]. Non-linear friction has also been considered using T-S fuzzy model in a small range near equilibrium position [TCW 08].

A methodology of **model reference adaptive fuzzy control (MRAFC)** is proposed to perform adaptive feedback linearization[SSS 89], i.e., to asymptotically cancel nonlinear functions in the system and place system poles into desired locations as specified by the reference model. The MRAFC scheme, which was first introduced in [KTJ 94], grew from ideas in conventional model reference adaptive control (MRAC) [BTH 92], [SYS 89], [AKJ 95] .The MRAFC scheme employs a reference model to provide closed-loop performance feedback for generating or modifying the knowledge base of fuzzy controller [ZLA 65]. A learning approach of combing MRAC with the use of fuzzy systems as reference models and controllers for control dynamical systems can be found in [LJR 93]. A hybrid approach by **combing fuzzy controller and neural networks for learning**-

based control is proposed in [SJT 96]. Furthermore, recent advances in the direction of using fuzzy control for MRAC can be found in [KOZ 94], [BRG 96]. The methodologies have been applied to a wide range of applications such as process control, cargo ship steering, and the control of robot manipulators. The basic architecture of our adaptive fuzzy controller is a class of fuzzy controllers [KTJ 93], called generalized multi linear fuzzy logic system (GMFLS), used in most fuzzy control systems [ZZY 93]. There exists an explicit function [KTJ 93] for GMFLS. In MRAFC scheme, the explicit form of the controller is used for both design and analysis.

There have been **several fuzzy-model-based approaches** concerning the stability of such nonlinear systems. Yurkovich and Widjaja [YKS 96] fully analyzed the control-engineering design procedures for an implementation of fuzzy-system concepts, and extend the linear quadratic fuzzy-based controller design to adapt to the changing system parameters in balancing control for the rotational inverted pendulum. Leung et al. [LFH 96] introduced a simple parallel distributed compensation (PDC) design technique for Takagi–Sugeno fuzzy logic controller (FLC) that can guarantee closed-loop stability. Wang et al. [WHO 96] also presented a design methodology for stabilization of a class of nonlinear systems based on Takagi–Sugeno fuzzy model and PDC control design, where stability analysis and control-design problems are reduced to linear matrix inequality problems.

Wei *et al.* [WEQ 95] presented a nonlinear control strategy by decomposing the control law into a sequence of steps. Chung and Hauser [CCC 95] proposed a nonlinear state controller that controls the cart position and the swinging energy of the pendulum at the same time. Lin and Sheu [LCE 92] proposed a hybrid-control approach that combines a FLC and a linear state feedback controller that stabilizes the linearized model of the inverted pendulum having a restricted travel. Zhao and Spong [ZHJ 01] applied a hybrid-control strategy, which globally asymptotically stabilizes the system for all initial conditions. Chatterjee et al. [CHD 02] proposed an energy based control law that swings up and stabilizes a cart-pendulum system with restricted travel and restricted control force in simulations as well as in real-time experiments.

In the past couple of decade, fuzzy logic control (FLC), as one of the most useful approaches for collecting human knowledge and expertise, has been the focus of numerous studies to control the plants that are mathematically poorly modeled or the model uncertainty in the dynamics is either unknown or impossible [SCY 94], [LCC 90], [LUK 94]. FLC design method is generally based on a fuzzy model that is constituted by a set of fuzzy *if–then* rules that maps inputs to outputs. A fuzzy model has excellent capability in complex and uncertain system description and is particularly suitable for modeling the nonlinear system by a set of fuzzy local models that are combined using a fuzzy inference mechanism corresponding to various operating points [TKT 85].

One of the important issues in constructing fuzzy model is the tradeoff between fitting the data of the trajectory of the nonlinear system and keeping a small amount of the fuzzy local models. Keeping a small amount of the fuzzy local models not only can prevent the model from over fitting but can provide significant computational and memory savings. However, this would not achieve the modeling objective and, thus, increase errors between the model output and the fitted data. Therefore, an intuitive approach when facing a nonlinear uncertain dynamic system is to increase the number of the fuzzy inference rules (e.g., fuzzy local models) to describe the dynamic system to any degree of accuracy [TKT 85], [JTA 94]. However, the fuzzy inference rules generally yields an exponential growth in the number of rules as inputs of the states are added to the system, quickly reducing performance to unacceptable levels and requiring more computation time and memory. Further, the inference rules for the fuzzy local models will typically contain a number of subjectively as well as empirically determined parameters and in most cases the nonlinearities existing in the dynamic system are not known.

To facilitate the control of the unknown dynamic systems, the authors [FNG 97] proposed a design method for constructing a global fuzzy controller from a set of local fuzzy controllers. However, when the controlled systems are with unknown parametric uncertainties and load disturbances, the design method can not guarantee stability of the closed-loop control system. Therefore, when the number of the fuzzy local models is significant, adaptive on-line control approach provides an interesting alternative. The advantage of the approach is that global stability analysis is simpler and a model structure can often be found with a minimum of modeling effort [SGM 86], [YWS 96].The adaptive

controller by adjusting the control parameters and their various practical applications have attracted much attention in the field of control engineering. In general, the external load disturbances always exist, although it is bounded. So, the controller without considering the disturbances can not stabilize the closed-loop control system. With dead-zone technique [PBB 82], [SSS 84], the controller will stop updating the control parameters when the identifier error is smaller than some threshold. Thus, it can prevent the estimated parameters from being infinity. However, the regulation error of the system will only be asymptotically bounded if large threshold is used, resulting in undesirable closed-loop performance. A strategy where a modified parameter update law to counteract the drift of parameter values into regions of instability in the absence of persistent excitation is designed on the basis of the (switching) -modification, is proposed by [IPA 86], [IPA 91], [IPA 96]. All control techniques have their individual characteristics. Hence, combining the merits of the adaptive control with that of the fuzzy control theories and then designing a new stabilizing controller will have better performance than that only based on control theory described in [SCY 94], [LUK 94], [TKT 85], [WLX 96].

Fuzzy control algorithms have been used in different configurations for a number of engineering applications. The majority of applications rely on an experts' knowledge of the system, analytical models of the system, offline system simulation in order to design the fuzzy parameters of the problem in hand. Then, the fuzzy controller can be implemented in software and run from some environment driving a DSP/IO board, loaded on general purpose or dedicated processor, or stored as parameters in an analog chip memory. In order to have an adaptive controller that is continuously running, an optimization algorithm has to be run-in parallel with feed-forward computation. This means, based upon the error measure of the state of the system, and a supervised optimization algorithm like Temporal Back Propagation (TBP) [JJS 95], [JJS 92], [NTR 95], [LKW 97], [YNB 97], [BSH 97] and Reinforcement Learning(RL) [BSH 92], [BAG 83], [LCT 94] to continuously adapt the parameters [WLX 97], [DRD 96], [JMM 93].

Fuzzy control provides a user-friendly formalism for representing and implementing the ideas to achieve high performance control system [PKM 94]. As a control target, the inverted pendulum is inherently unstable system. In order to balance the pendulum in an inverted position the pivot must be continuously moved to correct the falling pendulum.

This interesting control problem is fundamentally the same problem as that involved in rocket or missile propulsion [EDA 93].The rocket has to be balanced on its engine as it is accelerated. As the rocket tends to fall over the engine. Thrust must be deflected towards sideways to restore its course. Likewise, Bytronic Inverted Pendulum (IP) is a typical nonlinear dynamic unstable system.

Fuzzy control techniques have been used to control the second order IP system in presence of large but bounded disturbances [SZR 08]. Recently, various controlling techniques are implemented using fuzzy control, e.g.- Direct adaptive fuzzy control (DAFC), adaptive fuzzy sliding mode control (AFSMC) and adaptive neural fuzzy network control (ANFN) [WAR 08], [CHC 09].

Fuzzy swing up control (FSC) and Fuzzy sliding mode balance control (FSBC) has been used by Chang and Tao to control a Planatery gear type inverted pendulum (PIP). FSC upswings the pendulum quickly and FSBC stabilizes it in upright position where chattering effect is considerably eliminated [CHY 09]. Fuzzy control has also been used to control a two wheeled I P system [HUC 11].

In general, adaptive and robust control techniques provide solutions to the problem of modeling uncertainties. A good knowledge exists for adaptive control of linear systems, guaranteeing stability and robustness to uncertainties and disturbance. On the other hand, nonlinear adaptive control is rapidly developing and results in global stability and tracking results for relatively large classes of non linear systems [WLX 94], [IPA 96], [KHK 02], [KTM 95], [SYS 89].

The traditional approach to building system controllers requires a prior model of the system. The quality of the model, that is, loss of precision from linearization and/ or uncertainties in the system's parameters negatively influence the quality of the resulting control. At the same time, methods of soft computing such as neural networks or fuzzy logic possess non-linear mapping capabilities, do not require an analytical model and can deal with uncertainties in the system's parameters. Combined with the evolutionary

learning (such as genetic algorithms) these methods are capable of producing near-optimal controllers for a given control task. For example, genetic algorithms have been used to produce parameters of an optimized system controller such as the architecture and/or weights of a neural network controller [RNM 95],[DGD 93], rules and/or membership functions of a fuzzy controller [CMG 93], [CMM 97] and to obtain model equations [SHH 98], etc. The disadvantage of the Genetic Algorithms (GA) is that the process routinely produces solutions (parameter sets of a controller) that may rendr the controlled system unstable. A failure-free optimization method employing GA and a neural controller has been described in [SES 02].

Soft Computing:

The field of intelligent control research is around five decades old. Prof. L.A. Zadeh used the term soft-computing as collective methodologies, which fuse together fuzzy logic, neural network and genetic algorithms. The main feature of collective methodologies is their higher capability to tolerate imprecision, inaccuracies and partial truth.

The collective methodologies provide alterative solution to traditional modeling and design of control systems when system knowledge and dynamic models are incomplete. Collective methodologies have been successfully applied to many commercial products and industrial systems, where less accurate mathematical models are available. In addition, human knowledge or biologically motivated techniques are available to provide inference from limited available information [ZLA 94].

Kandadai and Tien [KRM 96] presented a fuzzy- architecture to automatically generate a fuzzy knowledge base by a pseudo-supervised learning scheme. Kawaji and Maeda[KWS 91] constructed a simple fuzzy controller that imbedded the cart position control into the pendulum angular control. Kyung and Lee [KHK 93] built a fuzzy controller, which rule base of 396 rules was derived from three neural networks. To stabilize a series-type double pendulum system, Muhammad, Odaka, Ogura, and Takahama [MMR 96] first inferred an intermediate force necessary for controlling each pendulum separately, and then inferred the actual force from the two intermediate forces. Terano, Murayama, Masui, and Tanaka

[TRN 86] designed a three mode fuzzy controller of four input items only for balancing the two pendulums.

In stabilization control of single and series-type double pendulum systems, four and six input items are separately necessary in order to cover the cart position control. Since the pendulum angular control(s) should be done first from intuition, the control priority orders have to be discriminated clearly. The SIRMs (Single Input Rule Modules) dynamically connected fuzzy inference model [YZN 97], [YZK 97] defines a SIRM and a dynamic importance degree for each input item. Because the single and double pendulum systems are similar control objects except of different dimensions, using a similar control architecture is preferable.

The earliest application of **neural networks** to the inverted pendulum task is that of Widrow and Smith [WDB 63] and Widrow [WDB 87]. A human controller who is able to stabilize an inverted pendulum is an alternative source of desired behavior, as demonstrated by Widrow and Smith [WDB 63], [WDB 87] and Guez and Selinsky [GZA 88]. Tolat and Widrow [TVV 88] provide a third demonstration of training with a human controller, with the novel use of feedback based on pixel values derived from a visual image of the pendulum location rather than the pendulum's state variables. Connell and Utgoff [CME 87] measured performance by the distance from the current state to the (0, 0, 0, 0) state, taking the action on each step that most reduced this difference. Rosen et al. [RBE 88] assumed a different type of knowledge, the knowledge that the inverted pendulum task is a failure-avoidance task. For avoidance tasks, longer trajectories through state space between failures are more desirable, so Rosen et al. identified actions that resulted in cycles in state space as the preferred actions.

The control of an inverted pendulum has been a long history as it is a good test bed for new control schemes. Almost all kinds of control strategies have been applied to this system. Cannon proposed a pole placement linear controller [CRH 99]. Mori proposed a control system that consists of two controllers, one is to swing up the pendulum, and the other is to stabilize the pendulum [MOS 76]. The latter is based on a state observer in case not all states are observable and is a simple proportional plus derivative feedback controller. Watts made a detailed study of the Linear Quadratic Regulator (LQR) design for the

inverted pendulum system. The Linear Quadratic Regulator design is based on a linear model of the system and a quadratic performance index that penalizes both the divergences of the output of the systems with respect to the desired values and the control effort. The result is a linear feedback of the state variable of the system multiplied by a set of gains. Watts showed that different weighting matrix would result in different linear quadratic regulators, and different regulators would generate different step response [WJW 84]. Dimeo suggested a very novel control scheme that combines a Linear Quadratic Regulator and a high frequency vertical oscillation of the pivot points of the inverted pendulum [DRM 94]. The upright position of the pendulum with a vertical oscillation of the pivot point was shown to be a stable equilibrium point. Nelson suggested a Linear Quadratic Regulator to control the pendulum angle to a limited range, and a neural network to further reduce the residual noise and enhance the control accuracy [NSJ 94]. Omatu proposed a control scheme that combines neural control and Linear Quadratic Regulator together. The output from a neural network is used to compensate the output from a Linear Quadratic Regulator to stabilize the pendulum when it is near the origin [OMS 94]. Deris proposed a control scheme that uses neural network to tune a Proportional Integral Derivative (PID) controller [DRS 95]. Genetic Algorithm (GA) was used to generate the initial values of the connection weights of the neural network and PID gains to avoid local minimum. Gordillo compared Linear Quadratic Regulator and Genetic Algorithm (GA) based approaches [GDF 97]. One conclusion from Gordillo is that GA is not very useful in design simple controller since many traditional method may work well Anderson proposed an optimal feedback control strategy in which a function is specified as the desired "flow" of trajectory, and a feedback control is computed by trying to drive the system in the direction opposite to the gradient of the function with respect to the state variables [AMJ 89]. The approach results in a nonlinear feedback control similar to the feedback control of a Variable Structure System (VSS) achieved through a sliding mode approach. Other neural network based approaches have also been proposed. Bouslama used a simple neural network to learn the input-output data from a fuzzy controller, and claimed that the neural network worked better than the original fuzzy controller [BSF 93]. Other researches emphasized on the learning ability of the neural network. They assumed that the system model is not clear, and a neural network is trained to control the inverted pendulum with

reinforcement learning [ACW 89] [YND 95]. In Eaton's approach, the weights of a neural network were optimized by GA with unknown pendulum dynamics [ETM 92]. Most of above approaches have been tested only on 2D inverted pendulum systems. Li used two fuzzy sliding mode controllers to control the cart position as well as the pole position [TZH 95]. Biega also successfully designed a dual neural network architecture to control both the pole and the cart position [BGV 96]. A hybrid scheme that combines fuzzy logic, deterministic rules and energy based control was also reported [DSM 95]. It was reported that this hybrid scheme could stabilize the pendulum from an arbitrary initial condition.

Using the inverted pendulum which is a typical nonlinear system, the research of control system that used human learning and knowledge has been done, for instance, there are research using a combination of neural network and GA [YMT 95], neural network [IKN 95] and fuzzy theory [MTK 96]. However, even human can control such a system, the development of the control method that improve the controller during the control execution remains as a difficult problem.

Due to its complex behavior, different versions of inverted pendulums have often been utilized to illustrate control methods, to investigate chaotic behavior, and to model systems of even greater complexity. The inverted pendulum is a link that is somehow maintained in a vertical position above the pin on which it is mounted. Several different configurations of the inverted pendulum are found in the literature. Anderson and Grantham [AMJ 89] demonstrate the Lyapunov optimal feedback control of a simple inverted pendulum, consisting of only a single rod mounted on a motor shaft. The control torque is limited to the torque necessary to hold the pendulum at 45^{O} from the vertical position. The controller achieves inverted vertical balance from the bottom position by "rocking" the pendulum out of potential energy wells. Latham [LPW 92] used a similar concept to develop algorithms for the control of bipedal walking robots. Gurumoorthy [GMR 92] describes the translational inverted pendulum or the "cart and pole" used for years as a benchmark for control theories. In this arrangement, the pendulum is maintained in a vertical position by moving the cart to which the pendulum is pinned laterally on a track. Furuta et a1.[FTK 92a] modified this configuration by adding a second link to the top of the first link and

placing the cart on an inclined path. In still another setup, Furuta et al. [FTK 98] replaced the cart with a horizontally rotating link. This apparatus allows only for tangential movement at the pin, effectively replacing the finite length of the original track with a track of infinite length.

The suggested method applies evolutionary learning to a neural controller in a subspace around the regulation point to ensure a failure free optimization process. Thus, due to the nature of the failure-free learning methodology, the optimized neural controller is capable of controlling the system in a relatively small region of state space, which may be a limiting factor for some practical applications of the optimized neural controller. This paper presents a hybrid controller that combines the benefits of an optimized neural controller and an LQG controller in a single system controller. The high quality of regulation process is ensured by application of the optimized neural controller, while the wide range of operation and stability of transient processes is provided by the LQG controller. A numerical model of an Inverted Pendulum (IP) served as the test bed for the development of the proposed hybrid controller. Utilization of a model instead of an actual system allowed expediting and simplifying the experimentation process. The performance and accuracy of the model was verified during the design of the LQG controller [DGD 98].

In the previous studies on control of inverted pendulums, Chow and Jacobson [CCK 72] developed a control strategy to stabilize an inverted pendulum with 2 degrees of rotational freedom about the base-point. The controller contained on–off perturbations and the base-point was allowed to move only in the vertical direction and in a prescribed manner. Levi [LVM 88] presented a topological explanation and a proof of stability of an inverted pendulum with a prescribed vertical base motion. Wu and Thornton-Trump [WUQ 97] proposed a discontinuous controller based on Lyapunov stability theory, for the control of 2 degrees of freedom pendulum subject to a general base-point motion in the vertical direction. Their work was later extended to design a smooth controller for the general three-dimensional base-point motion (Wu et al.), [WUQ 98] and rigorous proofs were given such that the controller guarantees the solution trajectory to be arbitrarily close to the upright position. In all the above studies, the controllers required knowledge of both the

movement of the base-point and the model of the pendulum. Nevertheless, the model developed by Wu et al. [WUQ 98] was further employed to study the control mechanisms of human trunk movements during normal walking (Wu et al.)[WUQ 98a]. These results showed that the model is capable of producing reasonable movements of human trunks and indicated the significance of this research. The challenge, however, is to further arrive at an alternative controller that: (i) guarantees the system stability, (ii) operates with minimum knowledge about the model and/or its parameters, and (iii) produces the responses similar to those from humans.

Itagaki et al. presented a design method for a reference filtering type, 2 degree of freedom control system with a structure of anti windup feedback control in order to overcome actuator saturation [TNI 08]. In case, if number of degree of freedom increases, the system becomes more under actuated. Chaturvedi et al. presented schemes for asymptotic stabilization of a 3-D pendulum where three rotational degrees of freedom are available [CHN 09], [CHN 09a]. Liu et al. proposed a scheme for non-local stabilization of a spherical inverted pendulum having four degrees of freedom [LIU 08].

The problem of the “inverted pendulum” is a classic topic in the field of automatic control, for it deals with an intrinsically unstable system, of relatively simple realization, whose dynamics is similar to those involved in tasks such as walking and the control of rocket thrusters [BAG 83], [JMI 90], [MWT 90]. Although a number of neural techniques have been used with simulated systems, little are those involving actual mechanical pendulums and hardware neural networks. Classical control theory tries to linearize the pendulum behavior close to the equilibrium position (a few degrees). This approach is able to maintain this position also with external forces trying to perturb the position. But out of the linearized interval the model looses precision and the system can become unstable or even the pendulum can fall down. The neural network doesn’t try to linearized the problem and shows good performances also quite far from the equilibrium position. In a first approach the researchers used the same equipment used to control the pendulum by means of a classical P.I.D. The researchers only replaced the control algorithm with neural network. In a second approach we used our Fast Prototyping Neural System (FPNS) [GSG 95] to

implement the neural net and to control the pendulum engine. The aim of the learning process is to train the neural controller to maintain the inverted pendulum balanced at the middle of the slide guides, and the specific algorithm used employs the "Back-propagation Through Time" (BTT) learning method, that allows introduction of the temporal factor in neural networks. A description of BTT can yet be found in the work of Rumelhart et al. [RMD 86]; a more detailed treatment is in [WPJ 90] and an application to a motion-control problem is in [NGD 89].

Many researchers have applied the neural network theory to control the inverted pendulum systems [IST 91], [WGJ 90], [ACW 89], [HAJ 93], [WMV 91], [SGM 91], [BAG 83]. Most of them were focused on the control problem of how to maintain an inverted pendulum in the neighborhood of an unstable equilibrium position. This kind of balancing problem is linear in nature. It cannot illustrate the robustness of those controllers for handling highly nonlinear problems. The grey system theory is a newly developed one and has widely been used in many fields [JGD 88]. With the development of grey theory and the microprocessor, the grey control theory is increasingly applied in the fields of engineering [LNM 89], [DGY 90], [JTL 92].

In recent years, there have been substantial theoretical advancements in the field of feedback control of dynamic systems under input and/or state-related constraints [BDS 95], [MYD 00]. In this connection, two main directions have emerged. On one side, nonlinear control design methodologies which explicitly take into account constraints in the design phase and result in control strategies that ensure, within given conditions, closed-loop stability, fulfillment of constraints and other control specifications. On the other side, control techniques finalized to modify on-line the command provided by a nominal linear controller so as to possibly prevent or handle constraint violation. Anti windup (AW) and command governor (CG) approaches are examples of this second class of techniques. Within the former class, problems of semi global or global stabilization of linear systems under input saturation have been addressed, e.g., in [SBA 96] and [LNZ 96] via a gain-scheduling ARE approach. Extensions to the simultaneous internal and external stabilization problem have also been reported in [HUP 98] and [SBA 00]. The theory of

positive invariant sets has been extensively used in solving regulation and tracking problems in the presence of constraints, model uncertainty and disturbances. In particular, polyhedral or ellipsoidal positive invariant sets have been considered, e.g. [HJC 91], [BCF 94], [VEI 96] and [ARJ 96]. Other contributions come from linear matrix inequality-based control design [BSS 94]. Finally, predictive control methodology [MDQ 00], [KSS 88], [KMV 96a] has been used to tackle tracking problems under constraints of various types. AW and CG strategies have some similarities but also conceptual differences [HNR 89], [KMV 94], [PNG 96], [KPN 98], [KMV 96] has been historically used for mitigating the "windup" phenomenon which occurs in control systems when input-saturation is present and the compensator contains dynamics.

The inverted pendulum is a classical benchmark for nonlinear control techniques, e.g. (Astrom and Furuta), [AKJ 00], and (Angeli), [AGD 01]. In particular, (Astrom, and Furuta), [AKJ 00] recently proposed an almost globally stabilizing strategy based on energy control for swinging up the pendulum. Starting from their results, we suggest here to resort to Model Predictive Control (MPC) to improve the control performance in terms of a cost function suitably selected by the designer. The idea is to add an extra term computed with MPC to the control signal provided by energy control. In so doing, it is possible to use this auxiliary signal to achieve some specific goals, such as the minimization of a prescribed cost. The natural setting of the problem and of the solution based on energy control is the continuous time, therefore also the MPC implementation proposed here is developed for continuous-time systems. However, it basically differs from the continuous time MPC algorithms for nonlinear systems previously published in the literature [CNH 98], [MYD 90]. In fact, these methods assume that the MPC law is continuously computed by solving at any (continuous time) instant a difficult optimization problem. This is impossible in practice, since any implementation is practically performed in digital form and requires a non-negligible computational time. The alternative method proposed here explicitly takes into account these constraints and is based on a truly digital approach relying on a continuous time problem formulation. In fact, it is assumed that the signal computed by MPC is piece-wise constant and with a limited number of free moves in the future. This leads to a discontinuous (with respect to time) control law which

preserves the stability of the overall system provided that the free tuning parameters of the MPC algorithm are properly chosen [MYD 00]. The “sampling time” between two successive solutions of the MPC problem must be selected to be greater or equal to the computational time required. However, it does not represent a critical parameter, since the auxiliary signal computed with MPC acts on an already almost globally stable system.

Inverted-Pendulum systems always exhibit many problems existing in industrial applications, for example, various nonlinear behaviors under different operation conditions, external disturbances, and physical constraints on some variables. Therefore, the task of real-time stabilization and tracking control of a highly nonlinear unstable inverted pendulum system has been a challenge for the modern control field. Recently, the control problems of inverted-pendulum systems have been intensively studied due to the challenging demand of fast and precise performance, where the mechanical structure can be divided into three main types in the literature. One is a pendulum with one degree of freedom (DOF) mounted on a base rotating around the vertical axis or actuated by a linear slide [ACW 89], [WLX 96], [MME 98], [HSJ 00], [MGA 99], [CCS 98], [HNJ 00]. The other is a double or triple inverted pendulum with more than one DOF actuated by a cart [MGA 95], [EKG 97], [TVA 99]. Another is a pendulum with two DOFs mounted on a robot with decoupled or weak coupled links [SPR 98], [KJH 99]. Huang and Huang [HSJ 00] proposed a grey prediction model combined with a proportional plus derivative controller to swing up the pendulum from a stable position to an unstable position and to bring its slider back to the origin of the track. However, the stability of this control scheme cannot be assured. Chen and Chen [CCS 98] introduced robust adaptive-control architecture for operating of an inverted-pendulum mechanism system to stable the pendulum at an upright position and to return the cart at its zero position. Tsachouridis and Medrano-Cerda [TVA 99] designed a robust computer control system in the sense of discrete-time H∞ theory for balancing and improving the steady-state performance of a triple inverted pendulum cart system. Though the stability of the control strategies in [CCS 98] and [TVA 99] can be guaranteed, some prior knowledge and constraints were required to ensure the stability of the overall system. Sprenger et al. [SPR 98] presented the balancing act of an inverted pendulum with a selective-compliant-assembly-robot arm

(SCARA) robot. The goal of this paper was to compensate for as many nonlinearities as possible and to design a robust linear control. However, the linear controller designed in [SPR 98] requires a sufficiently linearized system such that it cannot easily extend to control other servomechanisms. It is obvious that the control of an inherently unstable system is more difficult than the control of a stable one. Moreover, the control problem becomes more complex when more than one DOF is involved, e.g., the control objective of the pendulum angle and cart position with only one control force. In this study, a newly designed dual-axis inverted-pendulum system is introduced and the control object is to implement a suitable control scheme for stabilizing and tracking control of this nonlinear motor mechanism-coupled system. In the traditional inverted-pendulum mechanical system, DC motors have been widely used as high-performance drives. However, the mechanical commutators and brush assembly make dc motors more expensive than ac motors, and the use of mechanical commutators may produce undesired sparks[LHW 96], [KNR 01]. Recent advancements in magnetic materials, semiconductor power devices, and control theory have made the permanent-magnet (PM) synchronous motor drive play an important role in motion-control applications under the low to-medium power range. The desirable features of the PM synchronous motor are its compact structure, high air-gap flux density, high power density, high torque-to-inertia ratio, and high torque capability. Moreover, compared with an induction motor, a PM synchronous motor has such advantages as higher efficiency, due to the absence of rotor losses and lower no-load current below the rated speed, and its decoupling control performance is much less sensitive to the motor parameter variations. Thus, the PM synchronous motor is adopted as the actuator of the dual-axis inverted-pendulum mechanism in this paper. For the most part of the inverted-pendulum control in the published literature [ACW 89], [WLX 96], [MME 98], [HSJ 00], [MGA 99], [CCS 98], [HNJ 00], [MGA 95], [EKG 97], [TVA 99], [SPR 98], [KJH 99], actuator dynamics is typically excluded from the inverted-pendulum dynamic behavior to simplify the control design. However, actuator dynamics perform an important part of the complete inverted-pendulum dynamics, especially in the factors of high-velocity moment, highly varying loads, friction, and actuator saturation. Thus, there exist some interactions between inverted-pendulum and actuator dynamics that cannot be neglected. In this paper, the field-oriented control [LHW 96], [KNR 01], [WRJ 01] is used

in the design of the PM synchronous motor drive in order to achieve a fast four-quadrant operation and smooth starting and acceleration. In addition, the actuator dynamic of the field-oriented PM synchronous motor drive is included into the behavior of the inverted-pendulum mechanical structure. Sliding-mode control is one of the effective nonlinear robust control approaches since it provides fast system dynamic responses with an invariance property to uncertainties once the system dynamics are controlled in the sliding mode [SJE 91], [AKJ 95], [UKN 99]. The design procedure of sliding-mode control is first to select a sliding surface that models the desired closed-loop performance in the state variable space and then to design the control such that the system state trajectories are forced toward the sliding surface and stay on it. In the past decade, the sliding-mode control system has been widely used in various practical applications [WRJ 01], [BIC 00], [SKK 02] and [PKM 09].

Besides the cart type and furuta pendulums, the wheeled pendulums have been proved to be of much practical importance as they can play significant role in designing baggage transportation system [TKT 09], segway PT, gyrocar etc. Many researchers have done significant work for controlling a two wheeled inverted pendulum using neural network [JNS 08], [TSC 10], fuzzy control [HUC 11], sliding mode velocity control [HNJ 10] and adaptive output recurrent cerebellar model articulation controller (AORCMAC) [CHC 10].

4. Proposed Work

An inverted pendulum represents a device which is excellent for study of feedback control of an unstable system with different types of control strategies. It is containing of a number of nonlinearities and disturbances. Control of an inverted pendulum has been a benchmark example in demonstrating, motivating and developing various controller design techniques. The major challenge is to maintain robust performance with uncertain and unknown parameters.

The inverted pendulum system has been controlled using different conventional techniques such as PID control, sliding mode control, continuous state feedback control, energy control, asymptotic stabilization etc, but these techniques have not been proved to be very

effective to deal with the nonlinearities and uncertain behavior of the system. Recently, many researchers have used intelligent controlling techniques e.g. fuzzy based adaptive controller, ANN based control etc. These methods also have some limitations, such as performance of fuzzy controller depends upon appropriate acquisition of expert's knowledge and experience, which is one of the most difficult task and depends on many factors such as method of acquisition, mode of a person, level of expertise etc. ANN based controller is a black box process which doesn't clearly explains the system operation. Moreover, ANN training performance depends on its training algorithm and its parameters, accuracy of training data, their sequence of presentations, different mappings etc.

Therefore, a controller is required which may operate in totality to maintain the balance of inverted pendulum alongwith overcoming the unknown and uncertain nonlinearities and disturbances. It is well understood that the operating conditions of inverted pendulum system may vary due to uncertain nonlinearities which can't be foreseen through the theoretical studies. The on-line controlling approach has the advantage of direct control of nonlinear behavior of inverted pendulum system and yields more accurate model. Since the inverted pendulum model is intended to represent nonlinear and uncertain behavior under practical operating conditions, on-line data is very useful for deriving accurate inverted pendulum model.

Soft computing techniques work on adaptive learning techniques on the basis of practically available data. So, these techniques have been proved to be highly effective for controlling the nonlinear systems whose behavior can't be interpreted, such as inverted pendulum system.

The basic outline of the proposed work is as follows:

1. Computer Modeling and Simulation of inverted pendulum.
2. Development of an innovative control scheme using Quantum PSO/Quantum Evolutionary algorithm/Quantum Fuzzy approach/Generalized Neural Network and/or their combinations.
3. Validation of developed controller on benchmark problems.
4. Computer simulation of inverted pendulum with proposed control technique and comparison with standard controllers.
5. Real time control of inverted pendulum using proposed control technique.

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